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Continuous Rendezvous Games and Their Departure and Wait Times

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people attempting to rendezvous need to consider and make. What makes this game complex is that there are many stochastic factors such as road conditions involved. People cannot just start for the meeting place and arrive exactly when they want to. This means that a trivial solution case, I find pure strategy Nash equilibria characterized by two parameters, $\underline{s}$ and $\overline{s}$. These parameters respectively represent the earliest time at which players might depart and the latest time at which players might depart, according to the players' strategies. So the parameters describe how start time variation makes the players' departure times vary.

In subsection 4.1, for given departure times of the players, once the lower bounds on the player's value of the meeting are satisfied, players' values of the meetings can be arbitrarily higher in the pure strategy Nash equilibria. It is not necessary the player with the comparatively higher value of meeting that departs first in the pure strategy Nash equilibria. In the context of meetings involving the head of states, this means that the heads can deliberately depart late for a meeting and have the others wait for them.

In subsection 4.2, the Nash equilibria have low meeting probability because players do not always come nor wait for each other. If players compensate each other for arriving early and waiting, players might increase meeting probability and both their expected utilities. When monetary compensations are difficult to implement, non-monetary compensations such as agreeing that “the person who arrives late pays for the meal” can work in their place.

However, unilateral punishments for late arrival that go beyond compensation may decrease social welfare by harming the player who arrives earlier than the other player to avoid punishplayer's arrival and the cost of wait. In studying R&D, many papers have used the approach of nding the rms' optimal research decisions by comparing the hazard rate of invention and the cost of R&D. Kamien and Schwartz (1972) was the rst to analyze multi-player R&D models using hazard rates of inventions. However, in this paper, the rm considered only the hazard rate of invention for the composite rival and not its own hazard rate of invention. By doing so, the rm found the optimal invention time. In other words, the rm, unlike its rivals, is able to determine a invention time for its product.

All other subsequent papers I mention that study R&D using hazard rates instead have hazard rates of invention for all rms and nd game theoretic solutions by considering the rms' hazard rate with the costs of R&D. Loury (1979) and Lee and Wilde (1980) deal with a setting where every rm is identical. By comparing the hazard rate of invention and costs of R&D, rms nd the optimal investment in R&D to maximize expected pro ts[2] Reinganum (1983) analyzes an asymmetric setting with an incumbent rm and a challenger rm. This paper nds that the challenger invests more in R&D because the challenger has more to bene t from investment since it does not have current revenue. Doraszelski (2003) shows that when the rm's hazard rate of invention is a weakly increasing function of the rm's knowledge stock, the rm that is behind in R&D may invest more in R&D than the rm that is ahead.[3]

Many different causes can result in varying travel times (Kwon et al. 2011; Wong and Sussman 1973). Iida (1999) de nes travel time reliability as the probability of reaching the destination within a given time. The value of travel time reliability depends on the traveller's preferences. Polak (1987) and Senna (1994) derived expected utility formulas in which the value of travel time reliability was made explicit. Small (1982) was the rst to derive the Noland-Small equation.[4] The Noland-Small equation attempts to take into account the realistic considerations that go into scheduling a trip. Travellers want shorter travel times. They also do not want to arrive too early or too late. From the equation, I utilize the idea that the cost of travel time, cost of arriving early and the cost of arriving late can be separated and expressed additively. In the context of my model, the cost of arriving early becomes the cost of increased wait and the cost of arriving late becomes the loss from decreased meeting chance.

2. Choi (1991) is the seminal paper in which rms have the option to drop out from R&D. In my model, this dropping out is comparable to giving up on the meeting and abandoning the meeting place. Choi (1991) assumes that rms do not know their hazard rates of inventions. However, they observe the state of the other rm. Therefore, if the other rm makes partial progress on the invention, depending on the parameters, this can lead the rm to either drop out because of the technological gap or continue R&D because the rm now has reason to believe that the hazard rate of invention is hig002rms nohig002rmspartial p

3 Model

The rendezvous game has two people, player 1 and player 2, who make decisions about the meeting. Each person needs to decide by herself 1) whether she wants to come to the meeting at all, 2) when to depart for the meeting and 3) how long she waits for the other person at the meeting place. In making these decisions, people consider both the consequences of their own actions and the actions of the other person. While there is a benefit to a successful meeting, this comes at a cost of travelling time and potential waiting time. Leaving too early for the meeting place can mean the person has to wait longer for the other person. Leaving too late might cause the person to miss the other person entirely. People take these factors into consideration while choosing when to leave for the meeting.

3.1 Payoffs

To model the considerations of the player $i \in \{1, 2\}$, I use an expected utility framework following Morgenstern and Von Neumann (1953). When a player does not come to the meeting, her times to the meeting place, as random variables the realization of which players do not know before travelling.

By “continuous rendezvous games”, I mean that in this paper, for the most part, r_i follows a continuous distribution.[6] The codomains of the r_i's are R_+. The r_i's are independent of each other and the s_j's. If a CDF exists for r_i, the CDF is G_i and if the PDF exists for r_i, it is g_i.

3.3 Stages

This is a 2-stage sequential game. player i receives a private start time s_i, Then, player i chooses whether to depart for the meeting place. If she chooses to depart, she also chooses a departure time, $d_i \geq s_i$ and receives an arrival time $a_i = d_i + r_i$. d_i, r_i and a_i are also private. Given that player(s) i and j chooses (choose) to depart, d_i and d_j are conditionally independent. Later on, in specifying the distribution of a_i, $G_i(t) \equiv P(a_i \leq t)$ is used. If player i always comes to the meeting, $G_i(t)$ is a CDF of a_i and $G_i(t) = \int G_i(t - d_i) P(dd_i)$.[7] If $G_i(t)$ has a PDF, it is written as $g_i(t)$. The follwing is the specification of the stages, which is depicted in figure 1.

- Pre-game Setup
  1. Nature assigns each player i a random start time $s_i \geq 0$.
- Simultaneous Actions in Stage 1
  1. Each player decides on whether they will travel to the meeting place.
  2. Each player i who decided to travel decide the time at which they will depart for the meeting place. This time is called the departure time d_i or s_i.
- Simultaneous Actions in Stage 2
  1. Nature decides the r_i's for player who decided to travel in stage 1.
  2. After seeing their own $a_i = d_i + r_i$'s, each player i who decided to travel privately decides the time beyond which they will not wait and instead, abandon the meeting place. This time is called planned abandonment time, z_i, or player i who travels chooses $z_i \in [0, \infty]$, in other words, z_i is an element of the extended real line.
- Payoffs
  1. Players' payoffs are their expected utilities from the game. Given all the decisions of the two stages, the rendezvous game is played out in the following way. Players who decided not to come do nothing. Players who decided to come depart for the meeting place at d_i and realize travel time r_i. Now, their arrival time is $a_i \equiv d_i + r_i$. Given their arrival time, we also have their actionable abandonment time, $z_i \equiv \max\{a_i, z_i\}$. This z_i is private information. The rendezvous is successful if and only if both players come and $\max\{a_1, a_2\} \leq \min\{z_1, z_2\}$. If the rendezvous fails, players who came leave the meeting space at z_i.

6. It takes on uncountably many values.

7. I will explain the integral, $G_i(t) = \int G_i(t - d_i) P(dd_i)$. If player i is to arrive no later than t, given d_i, player i's travel time must be no more than $t - d_i$. Hence, the integrand is $G_i(t - d_i)$. I integrate over all d_i.

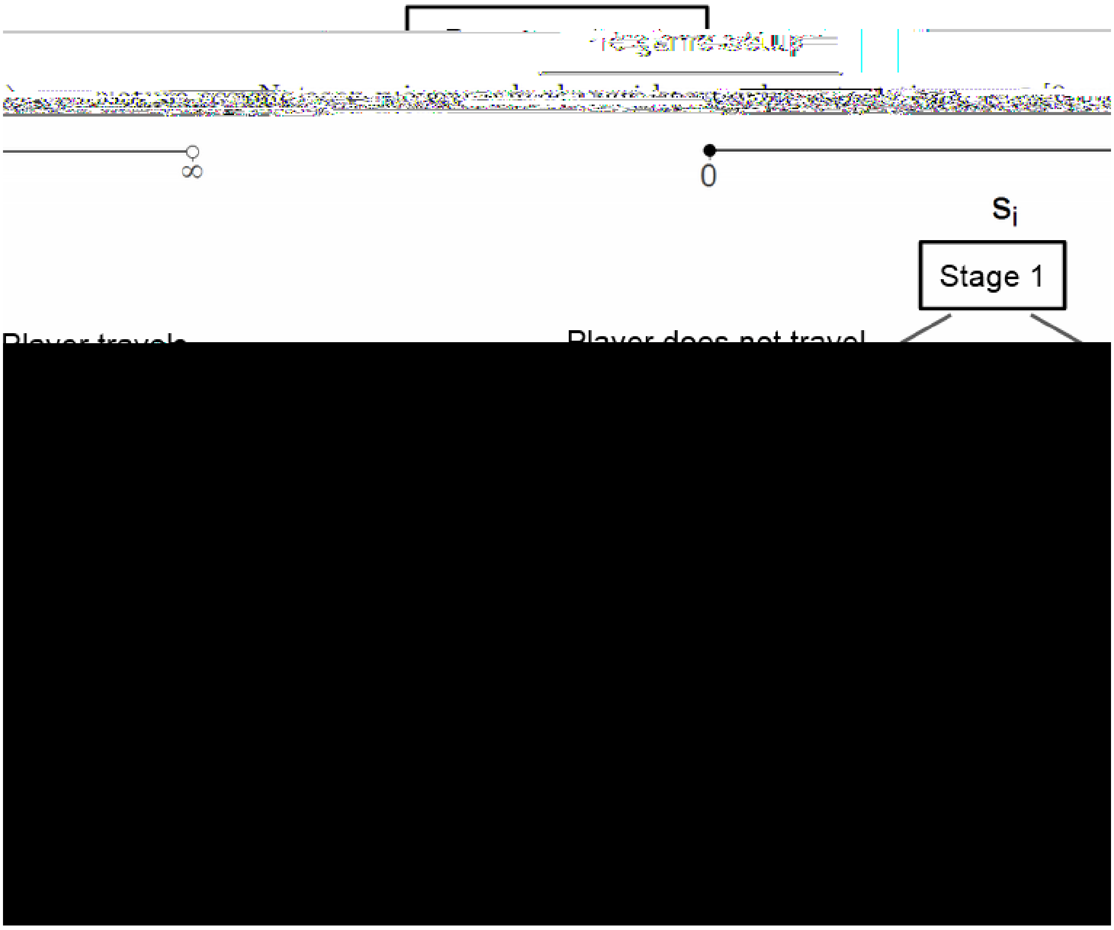

Figure 1: The stages of the game

For convenience, I define a random variable M the following way.

$$M = \begin{cases} 1 & \text{if } \max\{a_1; a_2\} \leq \min\{z_1; z_2\} \text{ (i.e. if rendezvous succeeds)} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

By this definition, E(M) becomes the probability of the players meeting.[8]

A noteworthy point is that the players do not decide on their wait times, w_i's, directly. In fact, players indirectly plan their wait times using their planned abandonment time. A player's actual wait time depends on when she and the other player arrive. The following is the exact formula for wait times.

$$w_i = \begin{cases} \max\{a_1; a_2\} - a_i & \text{if M=1} \\ z_i - a_i & \text{otherwise} \end{cases} \tag{3}$$

The logic for this indirection is similar to before. Once the players depart for the meeting place, there is nothing they can do to change the other player's arrival time. Furthermore, how long the players wait or when the players abandon the meeting place depends on the probability

8. By Lebesgue's dominated convergence theorem, M is Lebesgue integrable and equivalently E(M) is finite.

distribution of the other player's arrival. Given the player's departure and arrival time, the ex ante distribution of the other player's arrival tells the player when it is no longer worth it to wait for the other player. The player would set that time as the planned abandonment time.[9]

Actionable abandon time, z_i, exists to deal with cases where a player arrives after her planned abandonment time. In that case, the player would want to leave immediately unless her opponent is already at the meeting place. Then, the arrival time, not the planned abandonment time is when she abandons the meeting place should she fail to meet. The meeting happens if and only if both players arrive before any player would abandon the meeting place,

In this game, players can play mixed strategies. Thus, for a given arrival time, a_i, player i might have in nitely many optimal z_i

arrival is actually the value of the PDF of the other player's arrival). On the other hand, the cost of actionable desertion time is the conditional expectation the players haven't met, $E(1_{P(z_{-i}<a_i)}|a_i) + 1 - G_{-i}(z_i)$ times the marginal cost of waiting at the actionable desertion time. To state intuitively, in deciding whether to wait marginally more, the player considers the benefit given by multiplying the value of the meeting and the probability that the other player player will arrive during the marginal wait time. The player considers the cost given by multiplying the probability that the player actually has to wait and the marginal cost of wait.

I will explain this "probability" that the player actually has to wait in more detail. Obviously, the player only needs to wait if she hasn't met the other player yet. If she has, there is no wait. When the player has not meet the other player, she considers the two potential possibilities for why this has happened. The other player may have left early or he may have not come yet. To be elaborate, the first possibility, $E(1_{P(z_{-i}<a_i)}|a_i)$ is the conditional expectation that the other player already came and left the meeting place. The second possibility, $1-G_{-i}(z_i)$ is the probability that the other player will arrive in the future.

When player i's arrival time a_i is known and the probability that player -i abandoned the meeting place before this arrival time, $E(1_{P(z_{-i}<a_i)}|a_i)$ is 0, equation 4 can be restated as follows.

$$\frac{\partial E(u_i|d_i;a_i;z_i)}{\partial z_i} = g_{-i}(z_i)\bar{m}_i - (1-G_{-i}(z_i))\frac{\partial c_i(a_i-d_i;z_i-a_i)}{\partial z_i} \qquad (5)$$

Because equations 4 and 5 are difficult to analyze, I use the following formula. This formula u_i, z_i) has hap.559 -134549 Td 5me.

4 Results

4.1 Degenerate start times, s_i

Assumption 1. The following formulas hold for all $i \in \{1; 2\}$.

$s_i = 0$

If r_i exists, $r_i \sim U(0; 1)$.

$c_i(r_i; w_i) = r_i + w_i$

Assumption 1 specifies the start times, the s_i's, the travel times, the r_i's and the costs, the $c_i(r_i; w_i)$'s for this subsection. Here, players always start at time 0 and their travel time is distributed uniformly. Cost is the sum of travel time and wait time, w_i.

Assumption 2. Suppose that for any player i, fixed $a_i \geq 0$ and fixed $z_i \geq a_i$, $E(M|a_i; z_i) = 1$. Then, for any z that player i plays for a given a_i, $z \leq z_i$.

Assumption 2 caps how high planned wait time and actionable wait time can be for its cases. It states that for a given arrival time of $a_i \geq 0$, if waiting till time $z_i \geq a_i$ is sufficient to guarantee a meeting probability of 1, player i never waits be0 Td [(ne)25(v)15(er)-320(w)-3200J -309o8 0 Td [19 T02.

Proposition 1. Under assumptions 1 and 2, the following for some i is necessary and suf cient for a pure strategy Nash equilibrium with $E(M) > 0$. (In stating the following, I ignore 0 probability events and planned abandonment times for cases where the player has a 0 probability to wait)

(1) $\bar{m}_i \quad \max f^{(d_i \; d \; _i)^2}$

Note that by (1) and (2), $\bar{m}_2$ needs to satisfy both $\bar{m}_2 \geq m_2''(d_1;d_2)$ and $\bar{m}_2 \geq m_2'(d_1;d_2)$ while $\bar{m}_1$ only needs to satisfy $\bar{m}_1 \geq m_1'(d_1;d_2)$. $\bar{m}_2 \geq m_2''(d_1;d_2)$ comes from the requirement that player 2 weakly prefers not to delay departure. (For player 1, the condition that she weakly prefers to not delay departure is not binding.) $\bar{m}_2 \geq m_2'(d_1;d_2)$ and $\bar{m}_1 \geq m_1'(d_1;d_2)$ come for the requirement that player 2 and player 1 respectively weakly prefer to come to the meeting place. There is no upper bound on the players' values of meeting. Once the lower bounds on the players' values of meeting in (1) and (2) are met, players can have much higher values of meeting. Given $d_1 \geq d_2$, either player can value the meeting more highly in a pure strategy Nash equilibrium.

Since players always meet in the Nash equilibria of the proposition, $\bar{m}_2$ and $\bar{m}_1$ are respectively player 2 and 1's expected benefits in the Nash equilibria. $m_2'(d_1;d_2)$ and $m_1'(d_1;d_2)$ are respectively player 2 and 1's expected costs in the pure strategy Nash equilibria. Note that when $d_1 = d_2$, the expected costs are equal and $\bar{m}_2 \geq m_2''(d_1;d_2)$ is not binding. Proposition 1's (3) says $d_2 \leq d_1 < d_2 + 1$. There is no pure strategy Nash equilibrium with $d_1 - d_2 = 1$. This is because $d_1 - d_2 \to 1$, $m_2''(d_1;d_2) = \frac{(d_2-d_1)^2+1}{2(d_2+1-d_1)} \to \infty$.

Proposition 2. When $d_1 - 1 < d_2 \leq d_1$, the following holds.

(1) m_2', m_2'' and $m_1' + m_2'$ are [...]asing d

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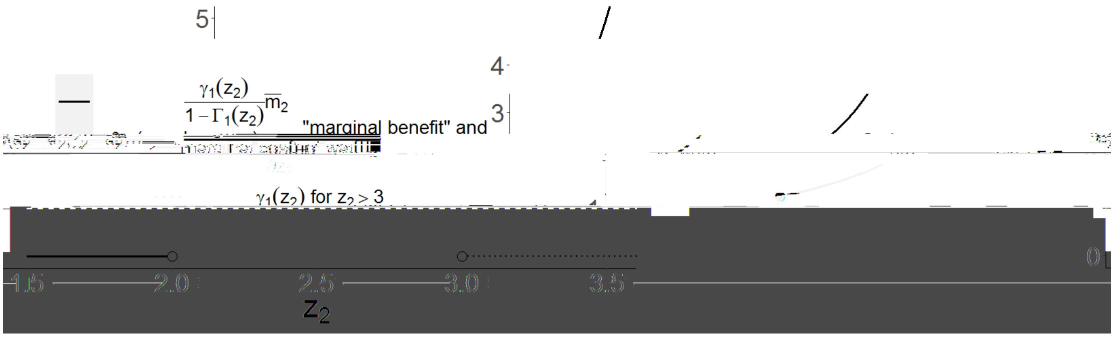

Figure 2: $\frac{g_1(z_2)}{1-G_1(z_2)}\bar{m}_2$, $g_1(z_2)$ and $\frac{\partial c_2}{\partial z_2}$ when $a_2 = 1.5$ for example 1

(2) $\bar{m}_1 = \frac{1}{2} + \frac{(d_2+1-d_1)^3}{6} \approx 0.52$

(3) $d_1 = 2$

(4) $d_2 = d_1 - 0.5 = 1.5$

(5) $z_1 = d_2 + 1 = 2.5$

(6) $z_2 = d_1 + 1 = 3$

Proof. The proof is by proposition 7 in appendix 2.

Now using hazard rate analysis, I will roughly explain why for specific arrival times, players find it optimal to wait till the other player arrives. For this, I use example 1 and figure 2 which is on this example. However, the explanation applies to any player in any Nash equilibrium of proposition 1. The figure draws functions with z_i, the actionable abandonment times on the X-axis. This will help me find the optimal z_i. Figure 2 depicts $\frac{g_{-i}(z_i)}{1-G_{-i}(z_i)}\bar{m}_i$, player $-i$'s hazard rate of arrival at z_i times player i's value of meeting $\bar{m}_i$, player $-i$'s density of arrival at z_i, $g_{-i}(z_i)$ and finally $\frac{\partial c_i(a_i-d_i; z_i-a_i)}{}$

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start time variation means that players may be unable to depart as early as they want to. The travel times, r_i's are also uniformly distributed for players who travel. For this subsection, proofs not found here are in appendix 3. The exact distributions are speci ed in the following assumption. This assumption for both players lays out the basic setting of the model.

Assumption 3.

s_i $U(0,1)$

If r_i exists, r_i $U(0,1)$.

When players are able to depart as early as they want to because of start time variation, they might depart later than they want to or not depart for the meeting. In order to describe these phenomena and strategies, I de ne two additional variables, $\underline{s}$ and $\bar{s}$ in the de nition below. The main focus of this subsections is symmetric Nash equilibria when the players face such constraints.[12]

De nition 2. $\underline{s} \in [0,1)$ is used for the earliest departure time by the players' strategies.

$\bar{s} \in (\underline{s},1]$ is used for the earliest departure time by the players' strategies.

In the following assumption I explain how exactly player's strategies depend on $\underline{s}$ and $\bar{s}$.

Assumption 4.

If s_i $\underline{s}$, $d_i = \underline{s}$.

If $s_i \in (\underline{s},\bar{s}]$, $d_i = s_i$.

If s_i

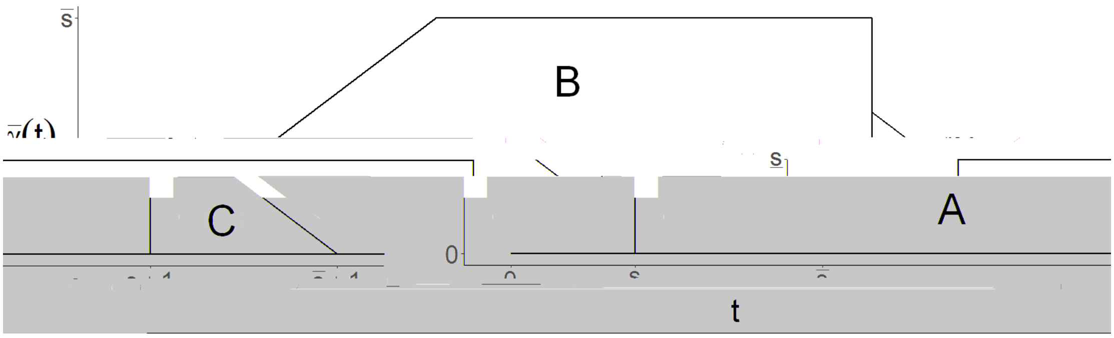

Figure 3: A PDF of definition 3 with polygons delineated

Definition 3.

(1) The following is a CDF.

$$\bar{G}(x) = \begin{cases} 0 & x \le \underline{s} \\ \frac{x^2 - \underline{s}^2}{2} & x \in [\underline{s}; \bar{s}] \\ \bar{s}x - \frac{\bar{s}^2 + \underline{s}^2}{2} & x \in [\bar{s}; \underline{s} + 1] \\ \bar{s} - \frac{(\bar{s} + 1 - x)^2}{2} & x \in [\underline{s} + 1; \bar{s} + 1] \\ \bar{s} & x \in [\bar{s} + 1; 10] \\ (1 - \bar{s})x + 11\bar{s} - 10 & x \in [10; 11] \\ 1 & x \ge 11 \end{cases}$$

(2) The following is a PDF of $\bar{G}(x)$.

$$\bar{g}(x) = \begin{cases} 0 & x < \underline{s} \\ x & x \in [\underline{s}; \bar{s}] \\ \bar{s} & x \in [\bar{s}; \underline{s} + 1] \\ \bar{s} + 1 - x & x \in (\underline{s} + 1; \bar{s} + 1] \\ 0 & x \in [\bar{s} + 1; 10) \\ 1 - \bar{s} & x \in [10; 11] \\ 0 & x > 11 \end{cases}$$

The CDF and PDF from the above definition lets me consider the players' arrival times as only have a positive probability of departing at $\underline{s}$ and she has a 0 probability of departing at any other point. The area of A is $\underline{s}$. The higher the $\underline{s}$, the greater its area.

Going back to figure 3, the B quadrilateral is from the player's departure after $\underline{s}$ for cases where the player arrives before $\underline{s}+1$ or at $\underline{s}+1$. The upward sloping edge of the B quadrilateral is due to the fact that if the player starts after $\underline{s}$ but not after $\bar{s}$, she departs immediately, adding on to the area of the B quadrilateral. Lastly, the C triangle is from cases where the player arrives after $\underline{s}+1$. Player only arrives after $\underline{s}+1$ when she departs after $\underline{s}$. This results in the downward sloping edge of the C triangle, which shows how the density of the players' arrival decreases after $\underline{s}+1$. In fact, unless $\underline{s}=0$, the PDF jumps downwards at $\underline{s}+1$. This decline in the PDF justifies why the players set their planned abandonment times to $\underline{s}+1$ and do not wait after $\underline{s}+1$.

The following definition introduces two functions used in concisely stating and proving the results of this subsection.

Definition 4.

$$\bar{i}(\underline{s},\bar{s}) \equiv \frac{6+2(\bar{s}+3)(\underline{s}+1-\bar{s})^3+3((\bar{s}-\underline{s})^2-2\underline{s})(\underline{s}+1-\bar{s})^2}{(12\bar{s}-6(\bar{s}-\underline{s})^2)(\underline{s}+1-\bar{s})}$$

$$\bar{w}(\underline{s},\bar{s}) \equiv \frac{1-\bar{s}}{\bar{s}-\underline{s}}+\frac{\bar{s}-\underline{s}}{2}$$

Proposition 3. Under assumption 3 for both players, assumption 4 for both players is a pure strategy Nash equilibrium if and only if $m_1 = m_2 = \bar{i}(\underline{s},\bar{s}) \le \bar{w}(\underline{s},\bar{s})$.

$\bar{i}(\underline{s},\bar{s})$ is the function used for the indifference condition, $m_i = \bar{i}(\underline{s},\bar{s})$. $\bar{w}(\underline{s},\bar{s})$ is the function used for the wait cap condition, $m_i \le \bar{w}(\underline{s},\bar{s})$. These two conditions are used to describe the symmetric pure strategy Nash equilibria of proposition 3. Under assumption 3 for both players, if and only if both conditions hold for both players, assumption 4 for both players is a Nash equilibrium. (In these Nash equilibria, by lemma 2 and formula 8, the meeting probability is $(\bar{s}-\frac{(\bar{s}-\underline{s})^2}{2})^2$.)

The first condition, $m_i = \bar{i}(\underline{s},\bar{s})$ means that player i's utility must be 0 when she departs at $\bar{s}$ and has a planned abandonment time of $z_i = \underline{s}+1$.[13] In other words, player i must be indifferent between departing at $\bar{s}$ to play $z_i = \underline{s}+1$ and not departing at all. Therefore, I call this the indifference condition. In figure 4, $E(m_i|d_i)$ and $E(c_i|d_i)$ respectively represent player i's benefit and cost when she departs at d_i and plays $z_i = \underline{s}+1$. (Figure 4 is drawn using proposition 9 in appendix 3.) In figure 4 and any Nash equilibrium of proposition 3, the two curves intersect at $d_i = \bar{s}$. Therefore, player i's expected utility at $d_i = \bar{s}$ is 0. So player i finds it optimal to come to the meeting when she starts before or at $\bar{s}$. It also means that she finds it optimal to not come to the meeting if she starts later. If m_i is higher, $E(m_i|d_i)$ increases at $d_i = \bar{s}$ and the intersection moves to the right. In this case, player i prefers to increase her latest departure time. If m_i is lower, $E(m_i|d_i)$ decreases at $d_i = \bar{s}$ and the intersection moves to the left. In this case, player i prefers to decrease her latest departure time.

The second condition, $m_i \le \bar{w}(\underline{s},\bar{s})$ is necessary for a player i who arrives to weakly prefer a planned wait time of $\underline{s}+1$ to a greater one. Hence, I call this the wait cap condition. I will explain this condition roughly using figure 5. Figure 5 applies the aforementioned technique of converting the distribution of player -i's arrival time, a_{-i} to follow definition 3 so that a CDF and a PDF exist to represent a_{-i}. Then, I can perform hazard rate analysis under the restriction of $z_i \in [a_i, \infty]$.

13. This condition also implies that a participating player i weakly prefers a planned wait time of $z_i = \underline{s}+1$ to a smaller one. This implication is shown by lemma 28 in appendix 3.

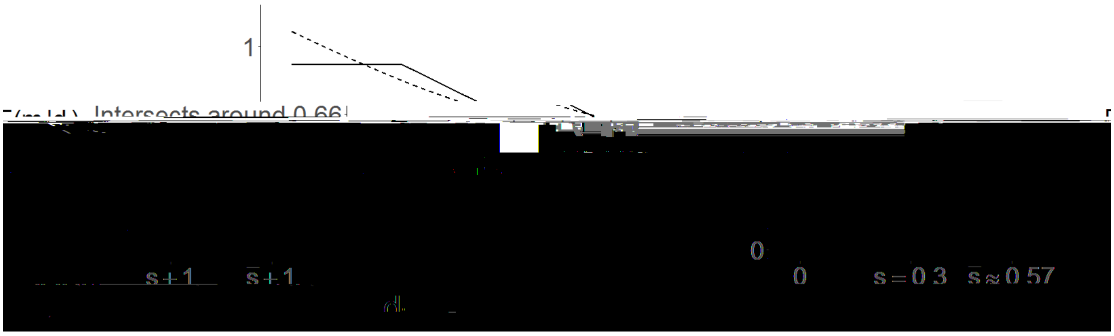

Figure 4: $E(m_i|d_i)$ and $E(c_i|d_i)$ when $\underline{s}=0.3$ and $\bar{s}\approx 0.57$

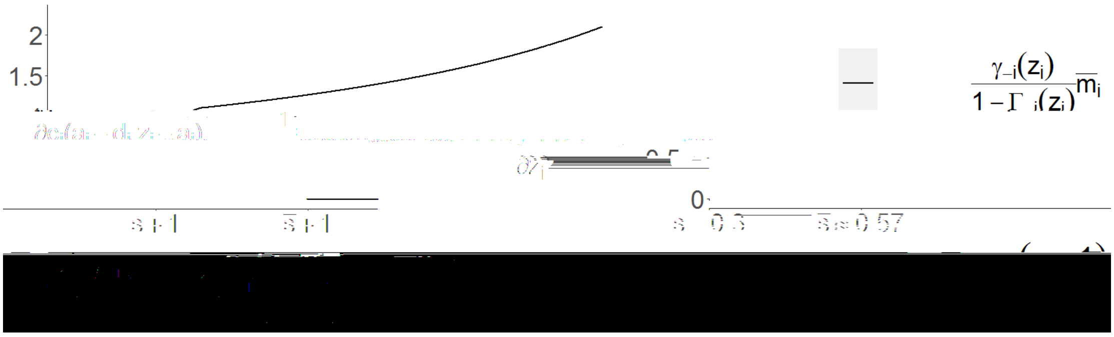

Figure 5: Hazard rate analysis using converted $\underline{p}$ when s

meeting engender low meeting chance in the Nash equilibria. Recall that the player's strategies are symmetrical in the Nash equilibria. If $\bar{s} < 1$ and start from a Nash equilibria with high $\underline{s}$. Here, for low values of meeting, players are willing to have $\bar{s}$ as their latest departure time. Now, refer to figure 4. For this $\underline{s}$ and $\bar{s}$, if players' values of meeting $\bar{m}_1$ and $\bar{m}_2$ are higher, this is no longer a Nash equilibrium because $E(m_i|d_i)$, player i's expected benefit for specific departure times would increase at $d_i = \bar{s}$. Therefore, players prefer to deviate to a higher latest departure time and increase their departure probabilities.

Next, suppose the players change their departure strategy so that they have a lower $\underline{s}$ but the same $\bar{s}$. This is a more demanding departure arrangement. Refer to figure 4 again. For the original players' values of meeting, $E(m_i|d_i)$ is below $E(c_i|d_i)$ at $d_i = \bar{s}$. Therefore, players prefer to deviate to a lower latest departure time. In other words, because they do not find it worthwhile to adhere to such a demanding departure arrangement and "fall off" by reducing arrival probability. In order for them to find it worthwhile to adhere to the departure arrangement, their values of meeting must increase when $\underline{s}$ decreases.

5 Discussion

5.1 Hazard rates and waits

The hazard rate of the other player's arrival often plays a key role in a player's wait decision. I will explain this informally. Usually, by comparing $\frac{g_{-i}(z_i)}{1-G_{-i}(z_i)}\bar{m}_i$ to $\frac{\partial c_i(a_i-d_i, z_i-a_i)}{\partial z_i}$, the player can figure out the sign of the marginal utility of actionable wait time z_i. Here, $\frac{g_{-i}(z_i)}{1-G_{-i}(z_i)}$ is the hazard rate of the other player's arrival which represents how likely the other player is to arrive marginally given that she has not arrived yet. To use this comparison, if the other player arrives first, she needs to wait until the player arrives. To restate, in deciding to wait marginally, the player looks at her values oftimeher4 9(the)7parture th5lues ofth60.9091 Tf -250(departure)-2501F28 7.9

implies that people who value the meeting highly enough to travel to it are willing to wait for it. Therefore, in reality, people would be likely to plan to wait for a substantially long time.

5.2 Strategic complementarity of arrivals and planned waits

I will first explain the strategic complementary of arrivals. The model always has a trivial pure strategy Nash equilibrium where both player never come to the meeting place. Here, no player ever comes because the other player never comes. When the players' values of meeting are sufficiently high, this Nash equilibrium coexists with Nash equilibria where players come and meet with positive probability such as those in propositions 1 and 3.

Now I will discuss the set of Nash equilibria of proposition 3 using figure 6. Here, as shown in proposition 4, $\bar{s}$ is decreasing in the players' values of meeting, $\bar{m}_1$ and $\bar{m}_2$. This means that players' departure probability and meeting probability are decreasing in the players' values of meeting. In these Nash equilibria, $\bar{s} < 1$ is true and strategic complementary of arrival works to lesson the departure probability of both players. In other words, because a player does not always come in the Nash equilibria, the other player also chooses to not always come.

To see this in the figure, pick a point on the $\bar{i}(\underline{s}, \bar{s})$ line where $\bar{s} \in (\underline{s}, s]$. (Recall that the $\bar{i}(\underline{s}, \bar{s})$ line is where players are indifferent between departing for the meeting place at $\bar{s}$ and not departing) On this point, fix the values of the players values of meeting, $\bar{m}_1$ and $\bar{m}_2$ as $\bar{m}_1 = \bar{m}_2 = \bar{i}(\underline{s}, \bar{s})$. Here, there exists a Nash equilibria of proposition 3. For a higher $\bar{s}$, $\bar{m}_1$ and $\bar{m}_2$ are above the $\bar{i}(\underline{s}, \bar{s})$ line. This means that if any player deviates from the Nash equilibrium strategy to play a strategy where they depart even if they start later than $\bar{s}$, the other player will be willing to also depart even if they start later than $\bar{s}$. This demonstrates that both players are stuck at the Nash equilibria with low $\bar{s}$, arrival probability and meeting probability because the other player plays the strategy with the low $\bar{s}$.

5.3 Meeting values and departure times

In the Nash equilibria of proposition 1, the player who departs earlier is not necessarily the player who values the meeting more. Once the lower bound conditions of the proposition's (1) and (2) are met, the players' values for the meeting can be arbitrarily higher. Proposition 2 reveals that in these Nash equilibria, the comparatively earlier the player departs, the higher her expected cost. Therefore, players want to depart late and have the other player wait for them. Proposition 2's (1) shows that the sum of the players' expected costs is increasing in the absolute value of the difference in players' departure times. The player who departs earlier incurs excessive expected wait cost from a social welfare perspective and the more the players departure times differ, the lower the social welfare.

has to be on or above the $\bar{h}(\underline{s};\bar{s})$ line so that the players are willing to depart at $\bar{s}$. The figures show that this does not necessary require that $m_1 = \bar{m}_2$ be higher than the level at the Nash equilibria of the proposition. When $\underline{s}$, $\bar{s}$, $\bar{m}_1$ and $\bar{m}_2$ take on the values, the wait cap condition, $\forall i \in \{1;2\}; \bar{m}_i \geq \bar{w}(\underline{s};\bar{s})$ is violated so players prefer to wait beyond $\underline{s}+1$ when they arrive.

Another way to see this is to look at figures 3. In figure 3, increasing $\bar{s}$ also increases the settings, unilateral penalty provisions that go beyond estimated damages can result in a decrease in social welfare.

The Nash equilibria of proposition 1 applies to the supply chain setting in the following way. From the upstream's perspective, departure corresponds to the rm starting work on the project or the product contracted by the downstream. Arrival corresponds to the rm nishing the contracted work on this project or product. This can be delivery or installation of the product. Wait time corresponds to the time from the completion of the upstream's work to when the downstream rm actually makes use of what the upstream completed.

From the downstream's perspective, departure means the downstream begins preparations for making use of the upstream's project or product. This preparation can be making space in its shelves or warehouses to place the product. It can also be readying the environment for the upstream's work or the installation of the product. In other cases, the downstream might prepare parts or equipment it will use in conjunction with the upstream's product. Arrival means completion of the preparations. Wait time is the time from the completion of the preparations to when the downstream actually starts makes use of the product or project from the upstream rm.

The meeting succeeds when the downstream rm receives the upstream rm's project or product and starts to make use of it. For instance, if the downstream starts using the parts from the upstream rm in assembly, that corresponds to a successful meeting. If the downstream rm displays and starts selling the product it receives from the upstream rm, that also corresponds to a meeting.

Consider the following unilateral penalty. If player 2 arrives late, she pays player 1 but player 1 never pays player 2. Player 2 needs a high value of the meeting for her to depart comparatively early and pay the high expected cost. A high ne on player 2's late arrival, provides the incentive for player 2 to not delay departure. By having player 2 depart early and wait for player 1, player 1 extracts player 2's surplus. In a supply chain setting, player 2's value of the meeting would be mostly determined by the payment for the ful llment of the contract. For player 1, unlike raising this payment to lower player 2's departure time, raising player 2's ne for late arrival is costless and also guarantees that player 2 cannot depart comparatively late.

Liquidated damages can compensate players for their wait costs. As discussed in the pre-viousdraisingv44(pl,8(In)Td [(viousd)-p295(rm299(thys)(29(assemblyeir)-304(219(estts.)025(v)25(al)-216 when players value the meeting more and depart at earlier times, since both players expect the other player to arrive earlier, both will abandon the meeting place earlier than before.

Suppose players initially fix $\underline{s} > 0$ instead of $\bar{s}$. In this case, as proposition 4 and figure 6 show, in the set of these Nash equilibria, $\bar{s}$ is decreasing in the players' values of the meeting. Meeting probability, $\left(\bar{s} - \frac{(\bar{s}-\underline{s})^2}{2}\right)^2$ is increasing in $\bar{s}$. Therefore, higher values of meeting lead to a lower meeting probability in this case also. However, $\bar{s}$ cannot be $\underline{s}$ or smaller. This guarantees that when players initially fix $\underline{s}$, the infimum of meeting probability is $\underline{s}^2$.

One way people can move to a Nash equilibrium with high meeting probability is the following script. A person may start by asking the question of “When is a good time for you to meet?”. After the two people find a meeting time at which they can arrive with high reliability, they could promise that “We won't come early but we will wait moderately”.

When players are constrained by start time variation, this script can lead them to a Nash equilibrium of proposition 3 and a meeting probability greater than $\underline{s}^2$. By saying, “We won't

Appendix 1. Intermediate results and proofs

The lemmas and propositions that are stated and/or proven here are about the basic attributes of the game and are used elsewhere to derive other results. When any of the four equivalent conditions in the lemma below is satisfied, the players meet. Reformulating the meeting condition helps prove many other results.

Lemma 2.

$$\max\{a_1; a_2\} \le \min\{z_1; z_2\} \tag{7}$$

$$\Leftrightarrow$$

$$a_1 = a_2, a_1 \le a_2 \le z_1 \text{ or } a_2 \le a_1 \le z_2 \tag{8}$$

$$\Leftrightarrow$$

$$a_1 \le a_2 \le z_1 \text{ or } a_2 \le a_1 \le z_2 \tag{9}$$

$$\Leftrightarrow$$

$$a_2 \le z_1 \text{ and } a_1 \le z_2 \tag{10}$$

Proof. I will first prove (7) → (8). Suppose $a_1 = a_2$. The consequent holds. Using symmetry, suppose $a_1 < a_2$. $a_2 \le z_1 \Leftrightarrow a_2 \le \max\{a_1; z_1\} \rightarrow a_2 \le z_1$. Thus $a_1 \le a_2 \le z_1$.

Now I will prove (8) → (9). If $a_1 = a_2$, $a_1 = a_2 \le z_1$. Using symmetry, if $a_1 \le a_2 \le z_1$, $a_1 \le a_2 \le z_1 \le z_1$.

Now I will prove (9) → (7). Using symmetry, if $a_1 \le a_2 \le z_1$, $a_1 \le a_2 \le z_2$.

Equations 7, 8 and 9 are equivalent.

Now I will prove (7) → (10). If equation 7 holds, $\max\{a_1; a_2\} \le z_1$ and $\max\{a_1; a_2\} \le z_2$.

Now I will prove (10) → (9). Using symmetry, if equation 10 holds and $a_1 \le a_2$, $a_1 \le a_2 \le z_1$.

The following proposition is used in marginal analysis. The proposition's (1) is used to state the marginal expected utility of actionable abandonment time, z_i. The proposition's (2) is used to find the sign of the marginal expected utility of actionable abandonment time, Proof. Using symmetry, I say $i = 1$. Suppose c_1 is differentiable in w_1, g_2 exists and g_2 is continuous in d. Fix d_1, a_1, z_1 and $z_1' \geq z_1$ so that they are possible values and $\{z_1, z_1'\} \subseteq d$. The following definition slightly abuses notation.

$$\Delta(z_1'; z_1) \equiv E(u_1|d_1; a_1; z_1'; a_2) - E(u_1|d_1; a_1; z_1; a_2)$$

If $a_2 \leq a_1 \leq z_2$, $\Delta(z_1'; z_1) = 0$.
If $a_2 \leq z_2 < a_1$, $\Delta(z_1'; z_1) = -c_1(a_1 - d_1; z_1' - a_1) + c_1(a_1 - d_1; z_1 - a_1)$.
If $a_1 \leq a_2 \leq z_1$, $\Delta(z_1'; z_1) = 0$.
If $a_1 \leq z_1 < a_2 \leq z_1'$, $\Delta(z_1'; z_1) = \bar{m}_1 - c_1(a_1 - d_1; a_2 - a_1) + c_1(a_1 - d_1; z_1 - a_1)$.
If $a_1 \leq z_1 \leq z_1' < a_2$, $\Delta(z_1'; z_1) = -c_1(a_1 - d_1; z_1' - a_1) + c_1(a_1 - d_1; z_1 - a_1)$.

If g_2 exists, $P(a_1 = a_2) = 0$.

$$\begin{aligned}
&E(u_1|d_1; a_1; z_1') - E(u_1|d_1; a_1; z_1) = \bar{m}_1 P(a_1 \leq z_1 < a_2 \leq z_1') \\
&\quad - (P(a_2 \leq z_2 < a_1) + P(a_1 \leq z_1 \leq z_1' < a_2))(c_1(a_1 - d_1; z_1' - a_1) - c_1(a_1 - d_1; z_1 - a_1)) \\
&\quad - \int_{z_1}^{z_1'} (c_1(a_1 - d_1; x - a_1) - c_1(a_1 - d_1; z_1 - a_1)) g_2(x)\, dx \\
&= \bar{m}_1 \int_{z_1}^{z_1'} g_2(x)\, dx \\
&\quad - (P(z_2 < a_1) + 1 - G_2(z_1'))(c_1(a_1 - d_1; z_1' - a_1) - c_1(a_1 - d_1; z_1 - a_1)) \\
&\quad - \int_{z_1}^{z_1'} (c_1(a_1 - d_1; x - a_1) - c_1(a_1 - d_1; z_1 - a_1)) g_2(x)\, dx
\end{aligned}$$

By the fundamental theorem of calculus, $G'_{-i}(z_i') = g_{-i}(z_i')$ when the domain of z_i' is d for the differentiation. Therefore, by the Leibniz integral rule, for the same domain for differentiation,

$$\frac{\partial E(u_i|d_1; a_1; z_1')}{\partial z_1'} = \bar{m}_1 g_2(z_1') - (P(z_2 < a_1) + 1 - G_2(z_1'))\frac{\partial c_1(a_1 - d_1; z_1' - a_1)}{\partial z_1'}.$$

This proves (1). Now I will prove (2) with the fixed values from (1).

$$P(z_2 < a_1) + 1 - G_2(z_1) = P(z_2 < a_1) + P(z_1 < a_2) \tag{12}$$

Suppose $z_2 < a_1$ and $z_1 < a_2$. If $a_1 \leq a_2$, $a_1 \leq z_2$. If $a_2 < a_1$, $a_2 \leq z_1$. Thus $\{z_2 < a_1\}$ and $\{z_1 < a_2\}$ are disjoint sets.

$$\begin{aligned}
&P(z_2 < a_1) + P(z_1 < a_2) = \\
&\qquad P(\{z_2 < a_1\} \cup \{z_1 < a_2\}) = 1 - P(\{a_1 \leq z_2\} \cap \{a_2 \leq z_1\}) = 1 - E(M)
\end{aligned} \tag{13}$$

Here, the last equality is by lemma 2 and equation 10. By equations 12 and 13, I have the following.

$$P(z_2 < a_1) + 1 - G_2(z_1) = 1 - E(M|a_1; z_1) \tag{14}$$

Lemma 3. Suppose g_1 and g_2 exist.

$$E(m_i | d_i, d_{-i}, z_i, z_{-i}) = \bar{m}_i \left(\int_{d_i}^{z_i} G_i(x - d_i) g_{-i}(x - d_{-i}) \, dx + \int_{d_{-i}}^{z_{-i}} g_i(x - d_i) G_{-i}(x - d_{-i}) \, dx \right) \tag{15}$$

Proof. Using symmetry, I will prove for $i = 1$. $E(m_1 | d$

Since G_1 and G_2 have PDF's, they are absolutely continuous. Therefore, I can use integration by parts for the last equality.[24]

I now have $\int_{a_1<a_2} m_1\, dP = \int_{d_1}^{z_1} G_1(x-d_1) g_2(x-d_2)\, dx$. Recall that $m_1 = \bar{m}_1$ if and only if the meeting succeeds. Therefore, symmetry gives $\int_{a_1>a_2} m_1\, dP = \int_{d_2}^{z_2} G_2(x-d_2) g_1(x-d_1)\, dx$.

$$E(m_1) = \bar{m}_1 \left(\int_{d_1}^{z_1} G_1(x-d_1) g_2(x-d_2)\, dx + \int_{d_2}^{z_2} g_1(x-d_1) G_2(x-d_2)\, dx \right) \qquad (16)$$

The above lemma calculates the expected benefit of the game using integration. To understand lemma 3, I can refer to lemma 2 and formula 8. In a continuous setting like this one where the probability of the players meeting by arriving at exactly the same time is 0, I only need consider two scenarios of a successful meeting. $a_i \le a_{-i} \le z_i$ includes the scenario where player i comes and player $-i$ comes after player i but before player i abandons the meeting place. $a_{-i} \le a_i \le z_{-i}$ includes the scenario where player $-i$ comes and player i comes after player $-i$ but before player $-i$ abandons the meeting place. I assume $a_i \ne a_{-i}$ for now and

If formula 17 holds, I have the following.

$$\forall a_i \in [d_i, d_{-i}] : d_{-i}+1 \in z_i(a_i) \tag{18}$$

When $a_i \in [d_{-i}, d_i+1]$, lemma 16's (3) means following.

$$P(z_{-i} < a_i) = 0.$$

$$P(z_{-i} < a_i) + \frac{d_{-i}+1-a_i}{2} \leq \frac{1}{2} \leq \bar{m}_i$$

Therefore, the following holds by lemma 11.

$$\forall a_i \in [d_{-i}, d_i+1] : d_{-i}+1 \in z_i(a_i) \tag{19}$$

By lemma 12, I have the following.

$$\forall a_i \in [d_{-i}, \min\{2\bar{m}_i + d_{-i} - 1, d_{-i}+1\}) : d_{-i}+1 \in z_i(a_i) \tag{20}$$

Now, I will look into z_{-i}. When $a_{-i} \in [d_{-i}, d_i+1]$, lemma 16's (2) means following.

$$P(z_i < a_{-i}) = 0.$$

$$P(z_i < a_{-i}) + \frac{d_i+1-a_{-i}}{2} \leq \frac{1}{2} \leq \bar{m}_{-i}$$

Therefore, the following holds by lemma 11.

$$\forall a_{-i} \in [d_{-i}, d_i+1] : d_i+1 \in z_{-i}(a_{-i}) \tag{21}$$

If $a_{-i} \in [d_i+1, d_{-i}+1]$, $P(a_i \leq a_{-i} \leq z_i) = 1$ and by lemma 19, I have the following.

$$\forall a_{-i} \in [d_i+1, d_{-i}+1] : a_{-i} \in z_{-i}(a_{-i}) \tag{22}$$

$a_{-i} > d_{-i}+1$ is impossible.

By what I have figured out till now about z_i and z_{-i}. I know that when formula 17 is satisfied, the players find z_i and z_{-i} of the Nash equilibrium optimal.

Next, I will look at d_i. I will find player i's utility in the Nash equilibrium. In the Nash equilibrium, by lemma 2 and formula 9,

$$E(M) = 1. \tag{23}$$

The following is player i's cost in the Nash equilibrium.

$$\begin{aligned}
&E(c_i) = E(r_i) + E(w_i) = \\
&\frac{1}{2} + \int_{a_i \leq d_{-i}} a_{-i} - a_i \, dP + \int_{d_{-i} \leq a_i \leq d_i+1; d_{-i} \leq a_{-i} \leq d_i+1} \max\{0, a_{-i} - a_i\} \, dP \\
&+ \int_{d_{-i} \leq a_i \leq d_i+1; d_i+1 \leq a_{-i}} a_{-i} - a_i \, dP = \\
&\frac{1}{2} + \int_{d_i}^{d_{-i}} \int_{d_{-i}}^{d_{-i}+1} y - x \, dy\,dx + \int_{d_{-i}}^{d_i+1} \int_{x}^{d_i+1} y - x \, dy\,dx \\
&+ \int_{d_{-i}}^{d_i+1} \int_{d_i+1}^{d_{-i}+1} y - x \, dy\,dx = \\
&\frac{1}{2} + (d_{-i}+1-d_i)\frac{d_{-i}-d_i}{2} + \frac{(d_i+1-d_{-i})^3}{6} + (d_i+1-d_{-i})\frac{d_{-i}-d_i}{2} = \\
&\frac{1}{2} + \frac{(d_i+1-d_{-i})^3}{6} + d_{-i} - d_i
\end{aligned} \tag{24}$$

Therefore, in order for player i to weakly prefer coming to the meeting, the following condition is required.

$$\bar{m}_i \quad \frac{1}{2} + \frac{(d_i + 1 \quad d_{\ i})^3}{6} + d_{\ i} \quad d_i$$

Note that this condition makes the condition imposed by formula 17 unnecessary.

Fix the value of d_i in the Nash equilibrium as d. By lemma 7, $d_i < d$ is not optimal. By lemma 17's (1), $d_i > d_{\ i}$ is not optimal. In nding the optimal d_i, I only need consider d_i 2 [d; d d

Using the Leibniz integral rule, I now nd derivatives for the case where, in addition to the conditions above, $d_i < d_{-i}$ also holds.

$$\frac{\partial E(c_i)}{\partial d_i} = (z_i - d_i$$

If $d^0 < d_i + 1$, the following derivative exists.

$$\frac{\partial E(M)}{\partial d_{-i}} = -1 \tag{33}$$

$$\begin{aligned} E(c_{-i}) &= E(r_{-i}) + E(w_{-i}) = \\ &\frac{1}{2} + \int_{d_{-i} \le a_i \le d_i+1;\, d_{-i} \le a_{-i} \le d_i+1} \max\{0; a_i - a_{-i}\} dP = \\ &\frac{1}{2} + \int_{d_{-i}}^{d_i+1} \int_{x}^{d_i+1} y - x \, dy\, dx = \frac{1}{2} + \frac{(d_i + 1 - d_{-i})^3}{6} \end{aligned} \tag{34}$$

If $d^0 < d_i + 1$, the following derivative exists.

$$\frac{\partial E(c_{-i})}{\partial d_{-i}} = -\frac{(d_i + 1 - d_{-i})^2}{2} \tag{35}$$

Recall that $z_i = d^0 + 1$ here. By equations 32 and 34, in order for player -i to weakly prefer coming to the meeting, the following condition needs to be fulfilled.

$$\bar{m}_{-i} \ge \frac{1}{2} + \frac{(d_i + 1 - d_{-i})^3}{6} \tag{36}$$

If $d^0 < d_i + 1$, I have the following derivative by equations 33 and 35.

$$\frac{\partial E(u_i)}{\partial d_{-i}} = -\bar{m}_{-i} + \frac{(d_i + 1 - d_{-i})^2}{2}$$

If formula 36 is fulfilled, $\bar{m}_i \ge \frac{1}{2}$ and since $d^0 \le d_i$, the following holds.

$$\forall d_{-i} \in [d^0; d_i + 1] : \bar{m}_{-i} \ge \frac{(d_i + 1 - d_{-i})^2}{2}$$

In this case, player -i does not prefer to delay her departure.

Proof of Proposition 1.

Compare propositions 1 and 7. (1)–(3) from both propositions map to each other in order. Ignoring 0 probability events and z_j for cases where the player j has a 0 probability to wait, (4)–(6) of proposition 7 means (4) of proposition 1.

Proposition 8. In the Nash equilibria of proposition 1, the following properties hold.

$$\forall i \in \{1; 2\}; z_i \ge a_i : \frac{\partial c_i(a_i - d_i; z_i - a_i)}{\partial z_i} = 1 \tag{37}$$

$$\forall i \in \{1; 2\}; a_i \in [d_i; d_i + 1] : \frac{g_{-i}(z_i)}{P(z_{-i} < a_i) + 1 - G_{-i}(z_i)} = \frac{g_{-i}(z_i)}{1 - G_{-i}(z_i)}: \tag{38}$$

$$\forall a_2 \in [d_2; d_2 + 1] : \frac{g_1(z_2)}{1 - G_1(z_2)} = \begin{cases} 0 & z_2 \in [d_2; d_1) \\ \frac{1}{d_1 + 1 - z_2} & z_2 \in [d_1; d_1 + 1) \\ \text{does not exist.} & z_2 \ge d_1 + 1 \end{cases} \tag{39}$$

If $z_2 = d_1 + 1$, $g_1(z_2) = 1$. If $z_2 > d_1 + 1$, $g_1(z_2) = 0$.

$$\forall a_1 \in [d_1; d_1 + 1] : \frac{g_2(z_1)}{1 - G_2(z_1)} = \begin{cases} \frac{1}{d_2 + 1 - z_1} & z_1 \in [d_1; d_2 + 1) \\ \text{does not exist.} & z_1 \ge d_2 + 1 \end{cases} \tag{40}$$

If $z_1 = d_2 + 1$, $g_2(z_1) = 1$. If $z_1 > d_2 + 1$, $g_2(z_1) = 0$.

$$\frac{\partial E(M)}{\partial d_2} = \begin{cases} 0 & d_2 < z_1 - 1 \\ d_1 - d_2 - 1 & d_2 \in (z_1 - 1; d_1] \\ -1 & d_2 \in [d_1; z_1) \\ d_2 - d_1 - 1 & d_2 \in (z_1; d_1 + 1] \\ 0 & d_2 \ge d_1 + 1 \end{cases} \tag{41}$$

$$\frac{\partial E(c_2)}{\partial d_2} = \begin{cases} 1 & d_2 \le d_1 - 1 \\ \frac{(d_2 + 1 - d_1)^2}{2} - 1 & d \; \ldots \end{cases}$$

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Tf 5.999 1.636 Td [(+)]TJ/F76 10.9091 Tf 1J/F28 1.196
2 [

I already have the derivatives for the case when the domain is $d_2 \in [z_1 - 1; d_1]$ and it is a proper interval. I can use formula 26 and 28.

$$\frac{\partial E(M)}{\partial d_2} = d_1 - d_2 - 1$$

$$\frac{\partial E(c_2)}{\partial d_2} = -\frac{(d_2 - d_1)^2 + 1}{2}$$

For the case where the domain is $d_2 \in [d_1; z_1]$, I have the following equations.

$$E(M) = \int_{d_2}^{d_1+1} x - d_2\,dx + \int_{d_2}^{z_1} x - d_1\,dx = \int_{0}^{d_1+1-d_2} x\,dx + \int_{d_2}^{z_1} x - d_1\,dx$$

$$\frac{\partial E(M)}{\partial d_2} = -(d_1 + 1 - d_2) - (d_2 - d_1) = -1$$

$$E(c_2) = \int_{d_1+1 \geq a_2 \geq d_2+1} a_2 - d_2\,dP + \int_{a_1 \geq a_2; a_2 \geq z_1} a_2 - d_2\,dP + \int_{a_2 \geq a_1} a_1 - d_2\,dP + \int_{a_1 \geq a_2; z_1 \geq a_1 \geq d_1+1} d_1 + 1 - d_2\,dP =$$

$$\int_{d_2}^{d_2+1} x - d_2\,dx + \int_{d_2}^{z_1} (x - d_2)(x - d_1)\,dx + \int_{d_2}^{d_1+1}\int_{d_2}^{x} x - d_2\,dy\,dx + \int_{z_1}^{d_1+1} (d_1 + 1 - d_2)(z \ldots \int_{d_x}^{z_1} \ldots 2\,dx +$$

$$E(c_2) =$$

$$\int_{d_1+1}^{d_2+1} x - d_2 \, dx + \int_{d_2}^{d_1+1}\int_{d_2}^{x} x - d_2 \, dy \, dx + \int_{d_2}^{d_1+1} (d_1+1-d_2)(x-d_1)\, dx =$$

$$\int_{d_1+1-d_2}^{1} x \, dx + \int_{d_2}^{d_1+1} (x-d_2)^2 \, dx + \int_{d_2}^{d_1+1} (d_1+1-d_2)(x-d_1)\, dx =$$

$$\int_{d_1+1-d_2}^{1} x \, dx + \int_{0}^{d_1+1-d_2} x^2 \, dx + \int_{d_2}^{d_1+1} (d_1+1-d_2)(x-d_1)\, dx$$

$$\frac{\partial E(c_2)}{\partial d_2} =$$

$$d_1+1-d_2-(d_1+1-d_2)^2-(d_1+1-d_2)(d_2-d_1)+\int_{d_2}^{d_1+1}(x-d_1)\,dx =$$

$$\frac{1}{2}+\frac{(d_1-d_2)^2}{2}$$

If the domain is $d_2 \ge d_1+1$, $E(M) = 0$ and $E(c_2) = 0.5$.

$$\frac{\partial E(M)}{\partial d_2} = \frac{\partial E(c_2)}{\partial d_2} = 0$$

If the domain is $d_2 \le d_1 - 1$,

$$E(c_2) = \int a_1 - a_2 \, dP = \int_{d_2}^{d_2+1}\int_{d_1}^{d_1+1} y - x \, dy \, dx = d_1 - d_2$$

and

When the domain is $d_1 \in [d_2 + 1; z_2]$ and it is a proper interval, $E(M) = z_2 - d_1$ by lemma 2

Appendix 3. Results and proofs used in subsection 4.2

The above lemma deals with the distribution of player i's arrival time, a_i. It states $P(a_i \le x)$ and $\frac{\partial P(a_i \le x)}{\partial x}$. Definition 3 uses these to create definition 3's CDF and PDF.

Proposition 9. Under assumption 3 for both players and assumption 4 for player i, the following formulas hold for player i when $z_i = \underline{s} + 1$.

(1)

$$E(M|d_i) = \begin{cases} \bar{s} - \frac{(\bar{s}-\underline{s})^2}{2} & d_i \le \underline{s} \\ (\bar{s} - \frac{(\bar{s}-\underline{s})^2}{2})(\underline{s}+1-d_i) & d_i \in [\underline{s}, \underline{s}+1] \\ 0 & d_i > \underline{s}+1 \end{cases}$$

(2) If $d_i \le \underline{s}$,

$$\begin{aligned} E(w_i|d_i) = & \int_{d_i}^{\underline{s}}\int_{\underline{s}}^{\bar{s}} (y-x) y\, dy\, dx + \int_{d_i}^{\underline{s}}\int_{\bar{s}}^{\underline{s}+1} (y-x)\bar{s}\, dy\, dx + \\ & \int_{\underline{s}}^{\bar{s}}\int_{x}^{\bar{s}} (y-x) y\, dy\, dx + \int_{\underline{s}}^{\bar{s}}\int_{\bar{s}}^{\underline{s}+1} (y-x)\bar{s}\, dy\, dx + \\ & \int_{\bar{s}}^{d_i+1} \frac{\underline{s}+1-x}{2}\bar{s}(\underline{s}+1-x)\, dx + (1-\bar{s}+\frac{(\bar{s}-\underline{s})^2}{2})(\underline{s}-d_i+0.5). \end{aligned}$$

If $d_i \in [\underline{s}, \bar{s}]$,

$$\begin{aligned} E(w_i|d_i) = & \int_{d_i}^{\bar{s}}\int_{x}^{\bar{s}} (y-x) y\, dy\, dx + \int_{d_i}^{\bar{s}}\int_{\bar{s}}^{\underline{s}+1} (y-x)\bar{s}\, dy\, dx \\ & + \int_{\bar{s}}^{\underline{s}+1} \frac{\underline{s}+1-x}{2}\bar{s}(\underline{s}+1-x)\, dx \\ & + (1-\bar{s}+\frac{(\bar{s}-\underline{s})^2}{2})(\underline{s}+1-d_i)\frac{\underline{s}+1-d_i}{2}. \end{aligned} \tag{47}$$

If $d_i \in [\bar{s}, \underline{s}+1]$,

$$\begin{aligned} & E(w_i|d_i) = \\ & \int_{d_i}^{\underline{s}+1} \frac{\underline{s}+1-x}{2}\bar{s}(\underline{s}+1-x)\, dx + (1-\bar{s}+\frac{(\bar{s}-\underline{s})^2}{2})(\underline{s}+1-d_i)\frac{\underline{s}+1-d_i}{2}. \end{aligned} \tag{48}$$

If $d_i \ge \underline{s}+1$,

$$E(w_i|d_i) = 0.$$

Proof. (1) uses the fact that when $z_i = \underline{s}+1$, by lemma 2 and formula 8, the players meet when $a_i \le \underline{s}+1$ and $a_{-i} \le \underline{s}+1$.

(2) uses the fact that when $x \in (\underline{s}$ 2 s891 7.9 1.[(()]TJ/F79 10.9091 Tf 4.232 0 Td [(s)]TJ ET q 1 0 0 1 270.3

Example 2. Suppose player -i's arrival time follows definition 3 and that $z_i = \underline{s} + 1$.

(1) Under assumption 3, when d_i and a_i are given,

$$\frac{\partial c_i(a_i - d_i; z_i - a_i)}{\partial z_i} = 1.$$

(2) when a_i is given,

$$\frac{g_{-i}(z_i)}{E(1_{P(z_{-i} < a_i)} | a_i) + 1 - G_{-i}(z_i)} = \frac{g_{-i}(z_i)}{P(z_{-i} < a_i) + 1 - G_{-i}(z_i)}$$

(3) If $a_i \in [0; \underline{s} + 1]$, $z_i \to \infty$ and $\bar{s} < \infty$,

$$\frac{g_{-i}(z_i)}{P(z_{-i} < a_i) + 1 - G_{-i}(z_i)} \to \lambda_{-i}(z_i)$$

$1 - G_{-i}(z_i)$

i

Next, I will prove that if s exists, $\bar{s} - \underline{s} < \frac{1}{3}$.

$$\frac{d\left(\frac{1}{\bar{s}-\underline{s}} + \frac{\bar{s}-\underline{s}}{2}\right)}{d(\bar{s}-\underline{s})} = -\frac{1}{(\bar{s}-\underline{s})^2} + \frac{1}{2} < 0 \quad (49)$$

$\bar{w}(\underline{s};\bar{s}) = \frac{1}{\bar{s}-\underline{s}} + \frac{\bar{s}-\underline{s}}{2}$ is decreasing in $\bar{s} - \underline{s}$.

$$\frac{\partial(2(\bar{s}-\underline{s}) - (\bar{s}-\underline{s})^2)}{\partial(\bar{s}-\underline{s})} = \underline{s} + 1 - \bar{s} > 0 \quad (50)$$

$2(\bar{s}-\underline{s}) - (\bar{s}-\underline{s})^2$ is increasing in $\bar{s} - \underline{s}$.

Consider the case where $\bar{s} \geq 0.5$. This implies $\bar{s} \geq 0.5$.

$$\bar{i}(\underline{s};\bar{s}) = \frac{6 + 2(\bar{s}+3)(\underline{s}+1-\bar{s})^3 + 3((\bar{s}-\underline{s})^2 - 2\underline{s})(\underline{s}+1-\bar{s})^2}{(12\bar{s} - 6(\bar{s}-\underline{s})^2)(\underline{s}+1-\bar{s})} \quad (51)$$

$$\geq \frac{6}{(12\bar{s} - 6(\bar{s}-\underline{s})^2)(\underline{s}+1-\bar{s})^3} \ldots \frac{12}{12\bar{s} - 6(\bar{s}-\underline{s})^2} \ldots \frac{1}{\bar{s} - \frac{1}{8}} \ldots \frac{8}{7}$$

5 10.9091 Tf 11.387 0 Td[(-0(is)-250(not)-250(a)-250(Nash)- m [48 [57

Here, the first weak inequality uses equation 34.

$$\bar{w}(\underline{s};\bar{s}) = \frac{1}{\bar{s}-\underline{s}} + \frac{\bar{s}-\underline{s}}{2} = \frac{1-\bar{s}+\underline{s}}{\bar{s}-\underline{s}} + \frac{\bar{s}-\underline{s}}{2} - \frac{\underline{s}}{\bar{s}-\underline{s}} = \frac{1}{\bar{s}-\underline{s}} + \frac{\bar{s}-\underline{s}}{2} - \frac{\underline{s}}{\bar{s}-\underline{s}} - 1 \quad (52)$$

By formula 49, I have the following.

$$\bar{w}(\underline{s};\bar{s}) \leq 2 + \frac{1}{4} - \frac{\underline{s}}{\bar{s}-\underline{s}} - 1 \leq \frac{5}{4} - \underline{s} \quad (53)$$

When $\underline{s} \geq \frac{1}{4}$, formulas 51 and 53 mean that this is not a Nash equilibrium. Suppose $\underline{s} < \frac{1}{4}$.

$$12\bar{s} - 6(\bar{s}-\underline{s})^2 = 6(2\bar{s} - (\bar{s}-\underline{s})^2) = 6(2(\bar{s}-\underline{s}) + 2\underline{s} - (\bar{s}-\underline{s})^2) < 6 + 12\underline{s} < 9$$

1 0 0 q 2 0 T m 43.31 f 4.244 0 Td [(()]TJ/F28 1534 9.6501 84 Tf 7.852 0.484 436 [(w)] 0 J/F [illegible]

Combine the above result with $\frac{12}{12\bar{s} - 6(\bar{s}-\underline{s})^2}$ from formula 51.

$$\bar{i}(\underline{s};\bar{s}) > \frac{4}{3} \quad (54)$$

Formula 53 and inequality 54 means that this is not a Nash equilibrium.

[illegible] /F28 +d 00.984 Td [(<)]TJ/F769091.9091 Tf 39.17 7.385[(()]TJ/F76w 0 0 m 5.455 0 l3 T0O BT /F76 10.9091 Tf 164

Here, the first weak inequality uses equation 34. The penultimate weak inequality uses the fact that $\frac{6-3x-6\bar{s}}{6x+12\bar{s}}$ is decreasing in x and formula 50. Combine formulas 55 and 56. In a Nash equilibrium, the following holds.

$$\frac{13}{4} - 3\bar{s} > \frac{9}{12\bar{s} - \frac{2}{3}} + \frac{1}{11}$$

$$(\frac{13}{4} - 3\bar{s})(12\bar{s} - \frac{2}{3}) > 9 + \frac{1}{11}(12\bar{s} - \frac{2}{3})$$

Next, I will prove that when $\bar{r}(\underline{s};\bar{s})$ $\quad$ $\bar{w}(\underline{s};\bar{s})$, $\bar{i}(\underline{s};\bar{s})$ is decreasing in $\bar{s}$. By the quotient rule and equation 34, it is suf cient to show that the following inequality holds.

$$\frac{\frac{¶((12\bar{s} \quad 6(\bar{s} \quad \underline{s})^2)(\underline{s}+1 \quad \bar{s}))}{¶\bar{s}}}{6+2\bar{s}(\underline{s}+1 \quad \bar{s})^3+(6 \quad 6\bar{s}+3(\bar{s} \quad \underline{s})^2)(\underline{s}+1 \quad \bar{s})^2} > \frac{\frac{¶(6+2\bar{s}(\underline{s}+1 \quad \bar{s})^3+(6 \quad 6\bar{s}+3(\bar{s} \quad \underline{s})^2)(\underline{s}+1 \quad \bar{s})^2}{¶\bar{s}}}{(12\bar{s} \quad 6(\bar{s} \quad \underline{s})^2)(\underline{s}+1 \quad \bar{s})} \tag{60}$$

$$\frac{¶(6+2\bar{s}(\underline{s}+1 \quad \bar{s})^3+(6 \quad 6\bar{s}+3(\bar{s} \quad \underline{s})^2)(\underline{s}+1 \quad \bar{s})^2}{¶\bar{s}} = 6(\underline{s}+1 \quad \bar{s})^3 \quad 6\bar{s}(\underline{s}+1 \quad \bar{s})^2 \quad (12 \quad 12\bar{s}+6(\bar{s} \quad \underline{s})^2)(\underline{s}+1 \quad \bar{s}) < 0 \tag{61}$$

$$\frac{¶((12\bar{s} \quad 6(\bar{s} \quad \underline{s})^2)(\underline{s}+1 \quad \bar{s}))}{¶\bar{s}} = 12((\underline{s}+1 \quad \bar{s})^2 \quad (\bar{s} \quad \frac{(\bar{s} \quad \underline{s})^2}{2})) \tag{62}$$

Consider the case where $\bar{s} \quad \underline{s} \quad \frac{1}{4}$.

$$¶\,($$

$$\frac{12((\underline{s}+1 \quad \bar{s})^2 \quad (\bar{s} \quad \frac{(\bar{s} \quad \underline{s})^2}{2}))}{(12\bar{s} \quad 6(\bar{s} \quad \underline{s})^2)(\underline{s}+1 \quad \bar{s})} = \frac{2(\underline{s}+1 \quad \bar{s})}{2\bar{s} \quad (\bar{s} \quad \underline{s})^2} \quad \frac{1}{\underline{s}+1 \quad \bar{s}}$$

$$\frac{\frac{3}{2}}{\frac{1}{2}+2\underline{s} \quad \frac{1}{16}} \quad \frac{4}{3} \quad \frac{24}{39} \quad \frac{4}{3} = \frac{28}{39}$$

Here, the rst weak inequality uses $\frac{¶(2\bar{s} \quad (\bar{s} \quad \underline{s})^2)}{¶\bar{s}} > 0$. Therefore, a suf cient condition is the following.

$$(6+2\bar{s}(\underline{s}+1 \quad \bar{s})^3+(6 \quad 6\bar{s}+3(\bar{s} \quad \underline{s})^2)(\underline{s}+1 \quad \bar{s})^2) \quad \frac{28}{39} > 6(\underline{s}+1 \quad \bar{s})^3 \quad 6\bar{s}(\underline{s}+1 \quad \bar{s})^2 \quad (12 \quad 12\bar{s}+6(\bar{s} \quad \underline{s})^2)(\underline{s}+1 \quad \bar{s})$$

$$6(\underline{s}+1 \quad \bar{s})^3+6\bar{s}(\underline{s}+1 \quad \bar{s})^2+(12 \quad 12\bar{s}+6(\bar{s} \quad \underline{s})^2)(\underline{s}+1 \quad \bar{s}) > (6+2\bar{s}(\underline{s}+1 \quad \bar{s})^3+(6 \quad 6\bar{s}+3(\bar{s} \quad \underline{s})^2)(\underline{s}+1 \quad \bar{s})^2) \quad \frac{28}{39} \tag{63}$$

$$\frac{¶(6(\underline{s}+1 \quad \bar{s})^3+6(\bar{s} \quad \underline{s})^2(\underline{s}+1 \quad \bar{s}))}{¶(\bar{s} \quad \underline{s})} = \quad s \quad 3))$$ 3))Tf11118877.3310.90991 Tf 1359.04 f 1525 TF 28258790 Td [5[

\ (6$\bar{s}$ + s

I transform the above using $\frac{27}{8}$ 9 $\frac{28}{39}(\frac{54}{64}$ $\frac{54}{16}) < 0$.

$$6 \quad \frac{27}{64} + 6 \quad \frac{9}{16} + \frac{6}{16} \quad \frac{3}{4} > \frac{28}{39}(6 + 2 \quad \frac{27}{64} + \frac{3}{9} \quad \frac{9}{16})$$

Since the above inequality holds, the $s \; \bar{s} \quad \frac{1}{4}$ case is proven.

Now consider the case where $s \; \bar{s} < \frac{1}{4}$.

$$\frac{\P((\underline{s} + 1 \quad \bar{s})^2 + \frac{(\bar{s} \quad \underline{s})^2}{2})}{\P(\bar{s} \quad \underline{s})} = 3(\bar{s} \quad \underline{s}) \quad 2 < 0$$

Therefore,

$$(\underline{s} + 1 \quad \bar{s})^2 \quad (\bar{s} \quad \frac{(\bar{s} \quad \underline{s})^2}{2}) > \frac{19}{32} \quad \bar{s}.$$

If $\bar{s} \quad \frac{19}{32}$, by formulas 60, 61 and 62, the case is proven. If $s \; \bar{s} > \frac{19}{32}$, formulas 60, 61 and 62 give me the following suf cient condition.

$$12(\frac{19}{32} \quad \bar{s})\frac{6 + 2\bar{s}(\underline{s} + 1 \quad \bar{s})^3 + (\text{[illegible]} + 1 \quad \bar{s})^2}{(12\bar{s} \text{ [illegible] } \underline{s})^2)(\underline{s} + 1 \quad \bar{s})} >$$
$$6(\underline{s} + \text{[illegible]})^3 \quad 6\bar{s}(\underline{s} + 1 \quad \bar{s})^2 \text{ [illegible] } 12\bar{s} + 6(\bar{s} \quad \underline{s})^2)(\underline{s} + 1 \quad \bar{s})$$

$$6(\underline{s} + 1 \quad \bar{s})^3 + 6\bar{s}(\underline{s} + 1 \quad \bar{s})^2 + (12 \quad 12\bar{s} + 6(\bar{s} \text{ [illegible] } + 1 \quad \bar{s}) >$$
$$12(\bar{s} \quad \frac{19}{32})\frac{6 + 2\bar{s}(\underline{s} + 1 \text{ [illegible] } + (6 \quad 6\bar{s} + 3(\bar{s} \quad \underline{s})^2)(\underline{s} + 1 \quad \bar{s})^2}{\text{[illegible] } \bar{s} \quad 6(\bar{s} \quad \underline{s})^2)(\underline{s} + 1 \quad \bar{s})}$$

From the above, I use the following to tr[illegible]m the inequality.

$$\frac{\bar{s} \quad \frac{19}{32}}{\bar{s} \quad 0.5(\bar{s} \quad \underline{s})^2} \quad \frac{\bar{s} \quad \frac{19}{32}}{\bar{s} \quad 0.125} \text{ [illegible] } 75 < 0.5$$

$$12(\underline{s} + 1 \quad \bar{s})^4 + 12\bar{s}(ss \text{ [illegible]}$$

Proof of Proposition 3.

Necessity is is by proposition 4's (1) and (2). I will prove sufficiency. If $\bar{s} < 1$ and $\bar{m}_1 = \bar{m}_2 = \bar{i}(\underline{s};\bar{s}) - \bar{w}(\underline{s};\bar{s})$, by lemma 28, $\bar{m}_i > \frac{(\bar{s}-\underline{s})^3+3(\bar{s}-\underline{s})^2-3\bar{s}+6}{6\bar{s}-3(\bar{s}-\underline{s})^2}$ is satisfied. Then, lemma 21's (1) and lemma 25 show that assumption 4 for both players is a Nash equilibrium. Lemma 26 establishes that if $\bar{s} = 1$, $\bar{i}(\underline{s};\bar{s}) - \bar{w}(\underline{s};\bar{s})$ is violated.

Lemma 5. If $\bar{s} < 1$ and $\bar{s} - \underline{s} \le \sqrt{2-2\bar{s}}$, $\bar{i}(\underline{s};\bar{s}) > \bar{w}(\underline{s};\bar{s})$

Proof. Suppose $\bar{s} - \underline{s} = \sqrt{2-2\bar{s}+e}$ for $e \le 0$. $\bar{s} - \sqrt{2-2\bar{s}+e}$. From definition 4, I have the following.

$$s_p(\bar{s}) =$$
$$\left(\frac{1}{2} + \bar{s} - \frac{(\underline{s}+1-\bar{s})^3}{6}\right)\sqrt{2-2\bar{s}+e} - \left(2-2\bar{s}+\frac{e}{2}\right)\left(1-\sqrt{2-2\bar{s}+e}\right)\frac{\bar{s}+\underline{s}}{2}$$
$$\ge \left(\frac{1}{2} + \bar{s} - \frac{(\underline{s}+1-\bar{s})^3}{6}\right)\sqrt{2-2\bar{s}+e} - (2-2\bar{s}+e)\left(1-\sqrt{2-2\bar{s}+e}\right)\frac{\bar{s}+\underline{s}}{2}$$

$$\frac{s_p(\bar{s})}{\sqrt{2-2\bar{s}+e}} = \frac{1}{2} + \bar{s} - \frac{(\underline{s}+1-\bar{s})^3}{6}$$

By equation 48, I have the following equations.

$$E(w_i | d_i = \bar{s}, z_i = \underline{s} + 1; s_{-i}) = \int_{\bar{s}}^{\underline{s}+1} \frac{\underline{s} + 1 - x}{2}$$

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