3. Let N 0 be an unknown integer and let $X_1, \dots, X_n$ be a random sample from the distribution with

$$P(X_i = k) = \begin{cases} \frac{1}{2N+1} & ; \quad k = 0, \quad 1, \quad 2, \dots, \quad N; \\ 0 & ; \quad \text{otherwise}. \end{cases}$$

   (a) Show that $M = \max\{|X_1|, \dots, |X_n|\}$ is a su cient statistic for N.

   (b) Show that M is a complete statistic for N.

   (c) Determine the uniformly minimum variance unbiased estimator (UMVUE) for N. Simplify your answer!
   Hint for part (c). Determine constants a, b, c such that $a\,|X_1| + b\,[\![X_1 = 0]\!] + c$ is unbiased for N, where $[\![X_1 = 0]\!]$ is notation for the indicator function of the event $[X_1 = 0]$.

4. Suppose that X_1 is a random sample of size 1 from the distribution with probability density function

$$f(x; \ ) = \begin{cases} \frac{1}{2} & ; \quad \text{if } 0 < x \quad ; \\ \overline{2x^2} & ; \quad \text{if } x \quad ; \\ 0 & ; \quad \text{otherwise}. \end{cases}$$

   For which values of $0 < \ < 1$ does $H_0 : \ = 1$ versus $H_1 : \ 3$ admit a unique uniformly most powerful (UMP) test of size ? Specify the rejection region associated with each of those tests.

5. Let $0 < \ < \ $ be real constants and consider an M/M/1 queueing system with arrival rate and service rate , <u>which is initially empty</u>. Suppose that the server is turned ON and OFF according to the following rules:

   it remains OFF as long as the number of customers in the system is less than 2,

   it is turned ON as soon as the number of customers in the system becomes 2 and then remains ON until completely emptying the system of customers.

   Based on the above, please respond:

   (a) Model the number of customer in the system as a continuous time Markov chain with state space $\{0, 1', 1, 2, 3, \dots\}$, where $1'$ represents the con guration of having exactly one customer in the system while the server is OFF, and state 1 represents the same but while the server is ON. Represent the rate transition matrix of the chain as a directed graph with weighted edges.

   (b) Show that this system has a stationary distribution and determine it explicitly.

   (c) After a long time of operation, what is the probability that a new customer encounters the queue empty?