

1. (30 pts) Determine $\frac{dy}{dx}$ for each of the following.

(a) $y = \sin^4(x^3)$

(b) $x^2 + xy + y^3 = 4$

(c) $y = \frac{2x^2 + 1}{x \cos x}$ After fully differentiating, do not algebraically simplify your answer any further.

Solution:

(a)

$$\begin{aligned} \frac{d}{dx} \sin^4(x^3) &= 4 \sin^3(x^3) \frac{d}{dx} \sin(x^3) = 4 \sin^3(x^3) \cos(x^3) \frac{d}{dx} x^3 \\ &= 4 \sin^3(x^3) \cos(x^3) (3x^2) = \boxed{12x^2 \sin^3(x^3) \cos(x^3)} \end{aligned}$$

(b)

$$\frac{d}{dx} x^2 + xy + y^3 = \frac{d}{dx} [4]$$

$$2x + xy' + y + 3y^2 y' = 0$$

$$y'(x + 3y^2) = -(2x + y)$$

$$y' = \boxed{\frac{-2x - y}{x + 3y^2}}$$

(c)

$$\begin{aligned} \frac{d}{dx} \frac{2x^2 + 1}{x \cos x} &= \frac{x \cos x \frac{d}{dx} [2x^2 + 1] - (2x^2 + 1) \frac{d}{dx} [x \cos x]}{(x \cos x)^2} \\ &= \frac{x \cos x (4x) - (2x^2 + 1) (x \frac{d}{dx} [\cos x] + \cos x \frac{d}{dx} [x])}{(x \cos x)^2} \\ &= \boxed{\frac{x \cos x (4x) - (2x^2 + 1)(-x \sin x + \cos x)}{(x \cos x)^2}} \end{aligned}$$

2. (25 pts) Parts (a) and (b) are unrelated.

(a) The position function of Particle P is given by $s(t) = 2t + t^2$, where s is in meters, t is in seconds, and $t \geq 1$.

i. Find the particle's velocity function $v(t)$. Include the correct unit of measurement.

3. (23 pts) Parts (a) and (b) are unrelated.

(a) Find the equations of the tangent and normal lines to the curve $y = x^{3/2} - x^{1/2}$ at $x = 4$.

(b) Find all values of x on the interval $[0; \pi]$ for which the curve $y = \sin^2 x - \sin x$ has a horizontal tangent line.

Solution:

$$(a) y'(x) = \frac{3}{2}x^{1/2} - \frac{1}{2}x^{-1/2} = x^{-1/2} \left(\frac{3}{2}x - \frac{1}{2} \right) = \frac{x^{-1/2}}{2} (3x - 1) = \frac{3x - 1}{2\sqrt{x}}$$

$$y'(4) = \frac{11}{4}$$

$$y(4) = 4^{3/2} - 4^{1/2} = 8 - 2 = 6$$

$$\text{Tangent line: } y - 6 = \frac{11}{4}(x - 4)$$

$$\text{Normal line: } y - 6 = -\frac{4}{11}(x - 4)$$

$$(b) y'(x) = 2 \sin x \cos x - \cos x = \cos x(2 \sin x - 1) = 0$$

$$\cos x = 0 \implies x = \frac{\pi}{2}$$

$$2 \sin x - 1 = 0 \implies \sin x = \frac{1}{2} \implies x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\text{Therefore, } x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$$

4. (22 pts) Parts (a) and (b) are unrelated.

(a) Determine $f'(x)$ for the function $f(x) = \frac{1}{x+1}$ by using the **definition of derivative**.

You must obtain f' by evaluating the appropriate **limit**

In order for g to be differentiable at $x = 2$, g must also be continuous at $x = 2$.

$\lim_{x \rightarrow 2} g(x) = \lim_{x \rightarrow 2^+} g(x) = g(2)$ leads to the following:

$$\frac{3}{8} 2^3 = (2^2) + \frac{17}{2}(2) + c$$

$$3 = 4 + 17 + c$$

$$c = \boxed{10}$$