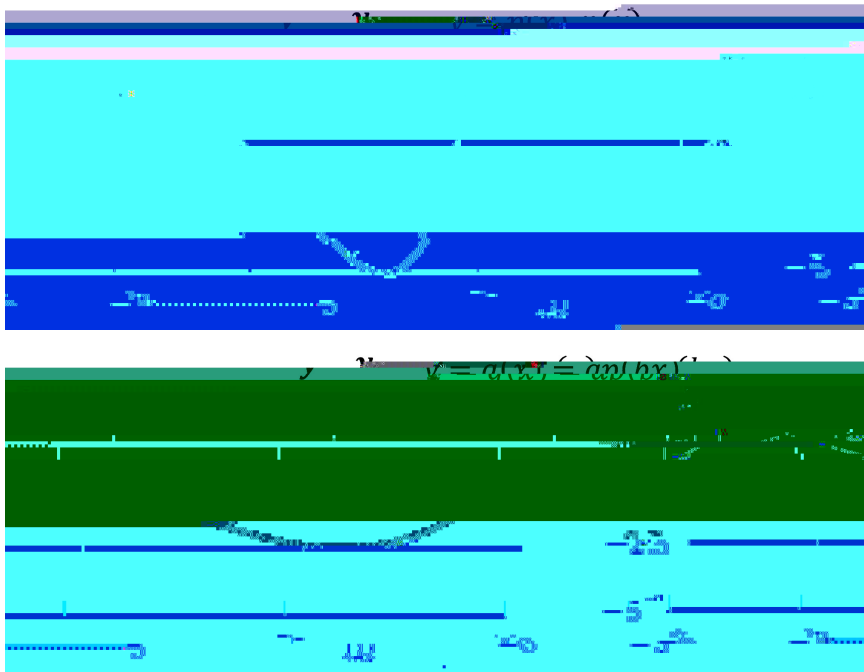


1. (20 pts) Parts (a) and (b) are not related.

(a) For  $f(x) = \frac{1}{x-1}$  and  $g(x) = \frac{1}{2-x}$

- (b) The graphs below depict the functions  $y = p(x)$  and  $y = q(x)$ , where  $q$  is a transformation of  $p$  of the form  $q(x) = ap(bx)$ . Find the values of  $a$  and  $b$ .



**Solution:**

The vertical difference between the maximum and minimum values of the curve for  $p(x)$  is  $3 - (-5) = 8$ , while the vertical difference between the maximum and minimum values of the curve for  $q(x)$  is  $1.5 - (-2.5) = 4$ . Therefore, the curve for  $q(x)$  has been constructed by applying a vertical contraction of a factor of 2 to the curve for  $p(x)$ . This implies that  $a = 1/2$

The horizontal difference between the endpoints of the curve for  $p(x)$  is  $5 - 3 = 2$ , while the horizontal difference between the endpoints of the curve for  $q(x)$  is  $10 - 6 = 4$ . Therefore, the curve for  $q(x)$  has been constructed by applying a horizontal expansion of a factor of 2 to the curve for  $p(x)$ . This implies that  $b = 1/2$

Note that  $q(x) = 0.5p(0.5x)$ .



$$(b) \lim_{x \rightarrow 2} \frac{\sqrt{x+1} - \sqrt{3}}{x^2 + x - 6}$$

**Solution:**

Begin by multiplying the numerator and the denominator by the conjugate of the original numerator expression.

$$\lim_{x \rightarrow 2} \frac{\sqrt{x+1} - \sqrt{3}}{x^2 + x - 6} = \lim_{x \rightarrow 2} \frac{\sqrt{x+1} - \sqrt{3}}{x^2 + x - 6} \cdot \frac{\sqrt{x+1} + \sqrt{3}}{\sqrt{x+1} + \sqrt{3}}$$

x

$$= \lim_{x \rightarrow 2}$$

3. (30 pts) Consider the rational function  $r(x) = \frac{3x^2 + 21}{x^2 - 4}$

- (c) Find the equation of each vertical asymptote of  $y = r(x)$ , if any exist. Support your answer in terms of your work in part (b).

**Solution:**

The finite value of  $\lim_{x \rightarrow 5} r(x) = \frac{9}{8}$  determined in part (b) indicates that there is no vertical asymptote at  $x = 5$ .

The infinite limits  $\lim_{x \rightarrow 3} r(x) = 1$  and  $\lim_{x \rightarrow 3^+} r(x) = 1$  were determined in part (b). Either one of those limits being infinite is sufficient to conclude that the line  $x = 3$  is a vertical asymptote of  $y = r(x)$ .

4. (20 pts) Parts (a) and (b) are not related.

(a) For what value of  $a$  is the following function  $u(x)$  continuous at  $x = 4$ ? Support your answer using the definition of continuity, which includes evaluating the appropriate limits.

$$u(x) = \begin{cases} \frac{x-4}{x^2-16} & ; x < 4 \\ \frac{1}{a-x} & ; x \geq 4 \end{cases}$$

**Solution:**

By the definition of continuity,  $u(x)$  is continuous at  $x = 4$  if  $\lim_{x \rightarrow 4^-} u(x) = \lim_{x \rightarrow 4^+} u(x) = u(4)$ .

$$\lim_{x \rightarrow 4^-} u(x) = \lim_{x \rightarrow 4^-} \frac{x-4}{x^2-16} = \lim_{x \rightarrow 4^-} \frac{x-4}{(x-4)(x+4)} = \lim_{x \rightarrow 4^-} \frac{1}{x+4} = \frac{1}{4+4} = \frac{1}{8}$$

$$\lim_{x \rightarrow 4^+} u(x) = \lim_{x \rightarrow 4^+} \frac{1}{a-x} = \frac{1}{a-4}$$

$$u(4) = \frac{1}{a-4}$$

Therefore,  $u(x)$  is continuous at  $x = 4$  if  $\frac{1}{8} = \frac{1}{a-4}$