1. (30 pts) Determine $\dfrac{dy}{dx}$ for each of the following.

(a) $y = \dfrac{\sin x}{2x + 1}$

Solution:

$$\frac{dy}{dx} = \frac{(2x+1)\,\frac{d}{dx}[\sin x] - \sin x\,\frac{d}{dx}[2x+1]}{(2x+1)^2} = \boxed{\frac{(2x+1)\cos x - 2\sin x}{(2x+1)^2}}$$

(b) $x^3 - y^3 = 5xy$

Solution:

$$\frac{d}{dx}[x^3 - y^3] = \frac{d}{dx}[5xy]$$

$$3x^2 - 3y^2 y' = 5(xy' + y)$$

$$3x^2 - 5y = (5x + 3y^2)y'$$

$$y' = \frac{dy}{dx} = \boxed{\frac{3x^2 - 5y}{5x + 3y^2}}$$

(c) $y = 4\cos^5(2x)$

Solution:

$$y = 4\cos^5(2x) = 4[\cos(2x)]^5$$

$$\frac{dy}{dx} = (4)(5)[\cos(2x)]^4 \cdot \frac{d}{dx}[\cos(2x)] = 20\cos^4(2x) \cdot -\sin(2x) \cdot \frac{d}{dx}[2x]$$

$$\frac{dy}{dx} = \boxed{-40\cos^4(2x)\sin(2x)}$$

2. (25 pts) The position value of a particle is given by $s(t) = t^2 - 4t^{1.5} + 4t$, where $t \geq 0$ is in seconds and position is in feet. For each of the following, be sure to include the correct unit of measurement.

(a) Find the particle's velocity function $v(t)$.

Solution:

$$v(t) = s'(t) = \frac{d}{dt}\left(t^2 - 4t^{1.5} + 4t\right) = 2t - (4)(1.5)t^{0.5} + 4 = \boxed{2t - 6t^{0.5} + 4 \text{ ft/s}}$$

(b) Determine the particle's speed at $t = \frac{9}{4}$ seconds.

Solution:

$$\left|v\left(\frac{9}{4}\right)\right| = \left|(2)\left(\frac{9}{4}\right) - (6)\left(\frac{9}{4}\right)^{0.5} + 4\right| = \left|\frac{9}{2} - (6)\left(\frac{3}{2}\right) + 4\right| = \left|\frac{9}{2} - \frac{18}{2} + \frac{8}{2}\right| = \left|-\frac{1}{2}\right| = \boxed{\frac{1}{2} \text{ ft/s}}$$

(c) Find the particle's acceleration function $a(t)$.

Solution:

$$a(t) = v'(t) = \frac{d}{dt}\left(2t - 6t^{0.5} + 4\right) = 2 - (6)(0.5)t^{-0.5} = \boxed{2 - 3t^{-0.5} \text{ ft/s}^2}$$

3. (25 pts) Parts (a) and (b) are unrelated.

(a) Find the equations of the tangent and normal lines to the curve $y = x^3 - 2x^2 + x + 10$ at $x = -1$.

Solution:

$y(-1) = (-1)^3 - 2(-1)^2 + (-1) + 10 = -1 - 2 - 1 + 10 = 6$

The point of tangency is $(-1; 6)$. 1

4. (20 pts) Parts (a) and (b) are unrelated.

(a) Determine $f'(x)$ for the function $f(x) = \sqrt{x+1}$ by using the **definition of derivative**. (You must obtain f' by evaluating the appropriate limit to earn credit.)

Solution:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sqrt{(x+h)+1} - \sqrt{x+1}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \cdot \frac{\sqrt{x+h+1} + \sqrt{x+1}}{\sqrt{x+h+1} + \sqrt{x+1}}$$

$$= \lim_{h \to 0} \frac{(x+h+1) - (x+1)}{h(\sqrt{x+h+1} + \sqrt{x+1})} = \lim_{h \to 0} \frac{h}{h(\sqrt{x+h+1} + \sqrt{x+1})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h+1} + \sqrt{x+1}} = \frac{1}{\sqrt{x+0+1} + \sqrt{x+1}} = \boxed{\frac{1}{2\sqrt{x+1}}}$$

(b) $\displaystyle\lim_{x \to 1} \frac{x^8 + 2x^5 - 3}{x - 1}$ represents the derivative of a certain function f at a certain number a.

i. Identify f and a.

Solution:

The definition of a derivative states that $f'(a) = \displaystyle\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$.

Since the given expression represents $f'(a)$ for some f and a, we have

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to 1} \frac{x^8 + 2x^5 - 3}{x - 1}$$

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