

1. The following are unrelated: (15 pts)

(a) Rewrite each of the following without the absolute value symbol:

i. $|j^2 - 6j|$

Solution:

Since $j > 3$, $j^2 > 6j$, $j^2 - 6j > 0$ so $|j^2 - 6j| = \boxed{j^2 - 6j}$

ii. $|j^{\sqrt{2}} - 2j|$

Solution:

Since $j^{\sqrt{2}} < 2$, $j^{\sqrt{2}} - 2 < 0$ hence $|j^{\sqrt{2}} - 2j| = \boxed{j^{\sqrt{2}} - 2}$ or $2 - j^{\sqrt{2}}$

(d) Add and simplify: $\frac{2}{9} + \frac{5}{12} + 7^0$

Solution:

$$\frac{2}{9} + \frac{5}{12} + 7^0 = \frac{14}{9} + \frac{5}{12} + 1 \quad (4)$$

$$= \frac{56}{36} + \frac{15}{36} + \frac{36}{36} \quad (5)$$

$$= \boxed{\frac{107}{36}} \quad (6)$$

(e) Simplify: $\frac{j}{2j} - \frac{7}{4j} - \frac{3j + j2j}{4j}$

Solution:

$$\frac{j}{2j} - \frac{7}{4j} - \frac{3j + j2j}{4j} = \frac{10 + 2}{8} \quad (7)$$

$$= \frac{12}{8} \quad (8)$$

$$= \boxed{\frac{3}{2}} \quad (9)$$

2. The following are unrelated. Leave your answers without negative exponents. (20 pts)

(a) $(5b^3)^2 7a^3 a^6$

Solution:

$$(5b^3)^2 7a^3 a^6 = (5^2)(b^3)^2 7a^3 \quad (10)$$

$$= 25b^6 7a^3 \quad (11)$$

$$= \boxed{175b^6 a^3} \quad (12)$$

(b) Simplify: $\frac{\sqrt[3]{32x^2}}{\sqrt[3]{2^4 \cdot 16}}$

Solution:

$$\frac{\sqrt[3]{32x^2}}{\sqrt[3]{2^4 \cdot 16}} = \frac{jxj \sqrt[3]{2 \cdot 16}}{\sqrt[3]{2^4 \cdot 4}} \quad (13)$$

$$= \frac{jxj4 \sqrt[3]{2}}{2 \sqrt[3]{2}} \quad (14)$$

$$= \boxed{2jxj} \quad (15)$$

(c) Simplify: $\frac{2(x^2 y^3)^3}{8x^3 y^{1-3}}$

Solution:

$$\frac{2(x^2 y^3)^3}{8x^3 y^{1-3}} = \frac{2x^6 y^9}{8x^3 y^{-2}} \quad (16)$$

$$= \frac{x^3 y^{27}}{4y^{\frac{1}{3}}} \quad (17)$$

$$= \boxed{\frac{y^{\frac{28}{3}}}{4x^3}} \quad (18)$$

(d) Multiply to rewrite as a polynomial: $\frac{1}{x-1} + 3 \cdot \frac{1}{x-1} - 3$

Solution:

$$\frac{1}{x-1} + 3 \cdot \frac{1}{x-1} - 3 = \frac{1}{x-1} \cdot 2 - 3^2 \quad (19)$$

$$= \frac{2}{x-1} - 9 \quad (20)$$

$$= \frac{2 - 9(x-1)}{x-1}$$

(b) Simplify the compound fraction: $\frac{\frac{3}{x^2} - \frac{1}{x}}{\frac{9}{x^2} - 1}$

Solution:

$$\frac{\frac{3}{x^2} - \frac{1}{x}}{\frac{9}{x^2} - 1} = \frac{\frac{3-x}{x^2}}{\frac{9-x^2}{x^2}} \quad (27)$$

$$= \frac{3-x}{9-x^2} \quad (28)$$

$$= \frac{3-x}{(3-x)(3+x)} \quad (29)$$

$$= \boxed{\frac{1}{3+x}} \quad (30)$$

(c) Factor by grouping: $9x^3 - 18x^2 - 4x + 8$

Solution:

$$9x^3 - 18x^2 - 4x + 8 = 9x^2(x-2) - 4(x-2) \quad (31)$$

$$= (x-2)(9x^2 - 4) \quad (32)$$

$$= (x-2)(3x)^2 - 2^2 \quad (33)$$

$$= \boxed{(x-2)(3x+2)(3x-2)} \quad (34)$$

5. Is $x = -2$ a solution of the inequality $x^3 - 2x < 2$

7. Solve each of the following equations. If there are no solutions write NO SOLUTIONS: (10 pts)

(a) $\sqrt[3]{8y+2} = y - 4$

Solution:

$$\sqrt[3]{8y+2} = y - 4 \tag{40}$$

$$\sqrt[3]{8y+2} = y - 4 \tag{41}$$

$$8y+2 = (y-4)^3 \tag{42}$$

$$y^3 - 11y + 28 = 0 \tag{43}$$

$$(y-7)(y-4) = 0 \tag{44}$$

$$y = 4; 7 \tag{45}$$

Plugging into the original equation, we find $y = 4$ to be extraneous. Hence $y = 7$

(b) Solve for h : $P = A + hdg$

Solution:

$$P = A + hdg \tag{46}$$

$$hdg = P - A \tag{47}$$

$$h = \frac{P - A}{dg} \tag{48}$$

8. Solve the following inequalities. Justify your answers by using a number line or sign chart if needed. Answers without full justification will not receive full credit. Express all answers in interval notation. (8 pts)

(a) $3x + 1 < 6$

Solution:

$$3x + 1 < 6 \tag{49}$$

$$3x < 5 \tag{50}$$

$$x < \frac{5}{3} \tag{51}$$

Hence the interval of solution is $\left(\frac{5}{3}; 7\right)$

$$(b) x^3 - 3x^2 = 0$$

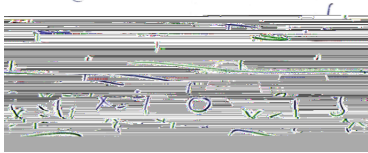
Solution:

We start by factoring the left hand side, and then make use of a number line/sign chart to choose the relevant interval of solution

$$x^3 - 3x^2 = 0 \tag{52}$$

$$x^2(x - 3) = 0 \tag{53}$$

Setting the left side equal to zero we get two values that make the left side zero: $x = 0$ and $x = 3$. Placing these on a number line and picking test values we obtain the following chart



Notice that $x = 0$ is a solution. Hence the solution is $0 \leq x < 3$.

9. Find all the solutions to the following equation, including the complex solutions (Hint: factoring will be important) $z^3 = 1$. (5 pts)

Solution:

$$z^3 - 1 = 0 \tag{54}$$

$$(z - 1)(z^2 + z + 1) = 0 \tag{55}$$

From which we conclude that $z = 1$ or

$$z = \frac{1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)} \tag{56}$$

$$= \frac{1 \pm \sqrt{-3}}{2} \tag{57}$$

$$= \frac{1 \pm \sqrt{3}i}{2} \tag{58}$$